Planar Multiport Quadrature-Like Power Dividers/Combiners

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Abstract—**A new class of planar, multiport power dividers/combiners is presented that is a generalization of the branch-line four-port 3-dB quadrature hybrid. They are suitable for combining an arbitrary number of identical reflection-type or transmission-type devices while maintaining a match at the input port. Experimental results are included for a four-way hybrid power divider/combiner.**

I. Introduction

A QUADRATURE 3-dB hybrid has the property of combining two identical reflection-type devices, e.g., p-i-n and IMPATT diodes, while maintaining a match at the input port without the need for a circulator, as shown in Fig. 1(a). Similarly, two such hybrids can be used for combining two identical transmission-type devices, e.g., FET's, while maintaining a match at the input port, even if the devices themselves are mismatched, as shown in Fig. 1(b). A branching cascade of these hybrids can also be used to achieve the same effect while combining n devices, where n is an integer power of 2. However, such a cascade becomes cumbersome for $n>4$.

In this paper, a new class of planar, multiport power dividers/combiners is presented that is capable of combining an arbitrary number of devices while maintaining a match at the input port. They are multiport generalizations of the familiar branch-line four-port 3-dB quadrature hybrid [1]. A black-box representation is shown in Fig. 2 where n reflection-type devices are being combined with one of these dividers/combiners. In the figure, P is the input port, Q is the output port, and ports 1 through n are device ports. Two such dividers/combiners can be used to combine transmission-type devices as was done in Fig. 1(b) with two 3-dB quadrature hybrids.

A multiport generalization of the quadrature hybrid that possesses the combining properties shown in Fig. 2 is described in [2]. However, it is nonplanar for $n>4$, and is not suitable for combining transmission-type devices because of its geometry.

In-phase n-way power dividers/combiners [3]–[6] can be used to combine reflection-type devices only through the use of a circulator since they do not possess a separate port equivalent to port Q in Fig. 2. However, they can be used to combine transmission-type devices while maintaining a match at the input port *if* unequal transmission-line segments are added to the device ports to stagger the phase appropriately.

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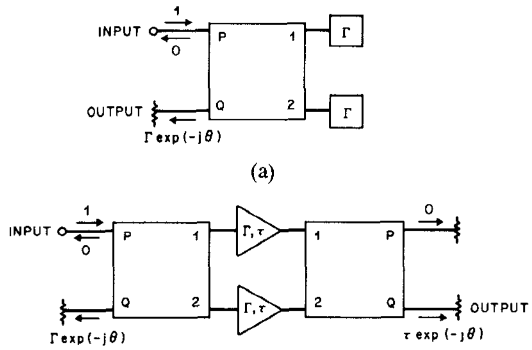

Fig. 1. The use of 3-dB quadrature hybrids for input-matched combining of two identical (a) one-port devices or (b) two-port devices.

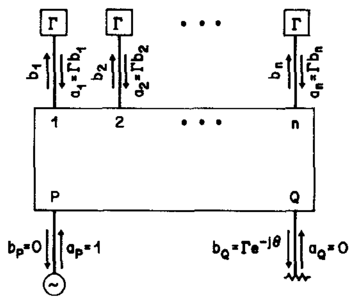

Fig. 2. A multiport generalization of the 3-dB quadrature hybrid used to combine n identical one-port devices. P is the input port, Q is the output port, and ports 1 through n are device ports.

II. Scattering-Matrix Formulation

A. Basic Results

Consider the $(n+2)$-port power dividing/combining network shown in Fig. 2. Let a_i and b_i be, respectively, the incident and reflected normalized wave amplitudes at port i, $i=1,2,\cdots,n,P,Q$. The normalized scattering-matrix equation of the network can be written as

$$\begin{bmatrix} \boldsymbol{b}_N \\ b_P \\ b_Q \end{bmatrix} = \boldsymbol{S} \begin{bmatrix} \boldsymbol{a}_N \\ a_P \\ a_Q \end{bmatrix} \tag{1}$$

where $\boldsymbol{b}_N=[b_1,b_2,\cdots,b_n]$ and $\boldsymbol{a}_N=[a_1,a_2,\cdots,a_n]$ are $n\times 1$ vectors, and $\boldsymbol{S}=[S_{ij}, i,j=1,2,\cdots,n,P,Q]$ is the $(n+2)\times(n+2)$ normalized scattering matrix.

As indicated on Fig. 2, let a wave of unit amplitude be launched at port P, a matched load be connected to port Q, and n loads with identical reflection coefficients, Γ, be connected to ports 1 through n. Thus

$$a_P=1 \quad a_Q=0 \quad \boldsymbol{a}_N=\Gamma \boldsymbol{b}_N. \tag{2}$$

For perfect combining with matched input, one must have

$$b_P=0 \quad b_Q=\Gamma \exp(-j\Theta) \tag{3}$$

where Θ is an arbitrary phase delay.

It is required to find the general form of $\boldsymbol{S}$ such that (1)–(3) are fulfilled for all values of Γ, and under the restrictions that the power dividing/combining network is reciprocal and passive, i.e., [1, pp. 148–149]

$$\tilde{\boldsymbol{S}}=\boldsymbol{S} \tag{4a}$$

$$\mathbf{1}-\tilde{\boldsymbol{S}}^*\boldsymbol{S} \text{ nonnegative definite} \tag{4b}$$

where the tilde represents matrix transposition, the asterisk represents complex conjugation, and $\mathbf{1}$ is the $(n+2)\times(n+2)$ identity matrix. It can be shown from (1)–(4) that $\boldsymbol{S}$ must assume the symmetric form

$$\boldsymbol{S}=\begin{bmatrix} \boldsymbol{R} & \boldsymbol{p} & \boldsymbol{q} \\ \tilde{\boldsymbol{p}} & 0 & 0 \\ \tilde{\boldsymbol{q}} & 0 & 0 \end{bmatrix} \tag{5}$$

with

$$\sum_{k=1}^{n} |p_k|^2=\sum_{k=1}^{n} |q_k|^2=1 \tag{6}$$

$$\sum_{k=1}^{n} p_k^2=\sum_{k=1}^{n} q_k^2=0 \tag{7}$$

$$q_k=p_k^* \exp(-j\Theta), \qquad k=1,2,\cdots,n \tag{8}$$

$$\boldsymbol{Rp}=\boldsymbol{Rq}=\mathbf{0} \tag{9}$$

where $\boldsymbol{R}=[R_{ij}\equiv S_{ij}, i,j=1,2,\cdots,n]$ is a symmetric $n\times n$ matrix, and $\boldsymbol{p}=[p_k\equiv S_{kP}=S_{Pk}, k=1,2,\cdots,n]$ and $\boldsymbol{q}=[q_k \equiv S_{kQ}=S_{Qk}, k=1,2,\cdots,n]$ are $n\times 1$ vectors.

B. Interpretation of the Results

The following interpretations of (5)–(9) are useful in subsequent developments.

a) The symmetry displayed by $\boldsymbol{p}$ and $\boldsymbol{q}$ implies that ports P and Q play a dual role, i.e., either of them can serve as an input port, and the other as an output port.

b) The four zeros in (5) signify that ports P and Q are matched and isolated from one another.

c) With the excitations and terminations given by (2) and Fig. 2, it follows from (1), (5), and (9), that

$$\boldsymbol{b}_N=\boldsymbol{p} \quad \boldsymbol{a}_N=\Gamma \boldsymbol{p} \tag{10}$$

for any value of Γ.

d) If $\boldsymbol{R}=\mathbf{0}$, which is consistent with (9), then ports 1 through n would be matched and isolated from one another. However, this is not implied by (9), and, in fact, not true in general, as will be seen later. Rather, (9) implies that the interactions among ports 1 through n cancel out when the incident wave-amplitude vector $\boldsymbol{a}_N$ at these ports is proportional to $\boldsymbol{p}$, as is the case in (10), or to $\boldsymbol{q}$, as would be the case had the excitation been applied to port Q.

e) With observations c) and d) in mind, (6) implies that all the power entering port P (or Q) emerges without loss from ports 1 through n. Moreover, (7) is responsible for the fact that waves reflected from these ports back to port P (or Q) cancel out, while (8) implies that all these waves add in phase at port Q (or P).

f) Using (4), one can show that if $\boldsymbol{p}$ and $\boldsymbol{q}$ satisfy (6)–(8), then (5) and (9), and, therefore, also (10), are automatically satisfied. Thus unless a particular matrix $\boldsymbol{R}$ is needed, the process of synthesizing a power divider/combiner reduces to requiring only $2n$ elements of its scattering matrix, namely the elements of $\boldsymbol{p}$ and $\boldsymbol{q}$, to satisfy (6)–(8), without paying any attention to the remaining elements or equations.

C. Solutions for $\boldsymbol{p}$ and $\boldsymbol{q}$

For $n=2$, (6)–(8) give a unique solution in which p_1 and p_2 have the same magnitude of $1/\sqrt{2}$, and are separated in phase by 90°. The same is true for q_1 and q_2. This, of course, is the case of the 3-dB quadrature hybrid. On the other hand, for $n>2$, an infinity of solutions exist. However, since the main application is in combining n identical devices, we restrict ourselves to the case of equal n-way power division. For simplicity, we also assume that, with an excitation at port P, the successive output waves $p_1, p_2, \cdots, p_n$ are separated in phase by the same delay angle, ϕ. Thus (6)–(8) give

$$p_k=n^{-1/2}\exp\{-j[\Theta_P+(k-1)\phi]\} \tag{11}$$

$$q_k=n^{-1/2}\exp\{-j[\Theta_Q+(n-k)\phi]\} \tag{12}$$

where $k=1,2,\cdots,n$, Θ_P and Θ_Q are arbitrary phase delays, and ϕ is given by

$$\phi=m\pi/n, \qquad m=1,2,\cdots, \text{or } n-1. \tag{13}$$

An arbitrary integer multiple of π can also be added to ϕ.

An equidivision, equidelay, n-way power divider/combiner satisfying (11)–(13) will be referred to as an $n(\phi)$ *divider/combiner*. Because of (13), the permissible combinations of $n(\phi)$ include 2(90°), 3(60°), 3(120°), 4(45°), 4(90°), 4(135°), $\cdots$, etc.

III. Planar, Lossless, Branch-Line Realizations

A. General Topology

The topology needed to realize the $n(\phi)$ dividers/combiners is by no means unique. The one employed here, which is shown in Fig. 3, has the advantages that it is planar, and that ports 1 through n lie on a straight line. The ladder-like structure in the figure represents the strip pattern of a microstrip or a stripline circuit, or the center conductor arrangement in a coaxial-line circuit. Thus all junctions are of the shunt type. With $k=1,2,\cdots,n$ and $l=1,2,\cdots,n-1$; Y_l, Y_l', and Y_k'' are characteristic admittances; Y_{0k}, Y_{0P}, and Y_{0Q} are port admittances; and B_k and B_k' are shunt susceptances. The electrical lengths of all horizontal lines are equal to ϕ, which is given by (13), and those of all vertical lines are equal to ψ, which is to be determined later.

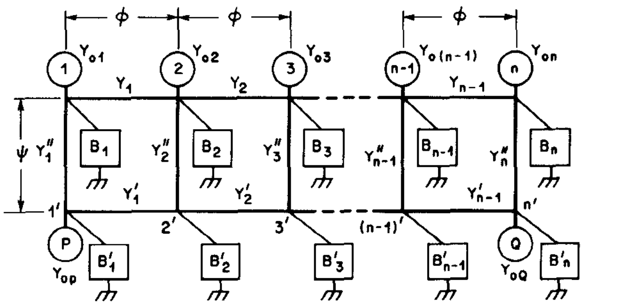

Fig. 3. The general planar topology used to realize $n(\phi)$ dividers/combiners. The ladder structure is a shunt connection of transmission lines, the squares are shunt susceptances, and the circles are ports.

The results given below can be applied to structures having series junctions by simply changing every Y to a Z, and every shunt B to a series X.

B. Specific Realizations

Let all admittances and susceptances be normalized to the admittances of ports P and Q, which will be assumed to be equal, and let the admittances of ports 1 through n be also equal. Thus

$$Y_{0P} = Y_{0Q} = 1 \tag{14}$$

$$Y_{0k} = Y_0, \qquad k = 1,2,\cdots,n. \tag{15}$$

It can be shown through the use of a straightforward, but lengthy, analysis that the circuit in Fig. 3 satisfies (11)–(13), if and only if $\theta_P = \theta_Q = \pi/2$, and if

$$\psi = \pi/2 \tag{16}$$

$$Y_l = Y_{n-l} = (Y_0/n)\left[l(n-l) + \sin^2(l\phi)/\sin^2\phi\right] \tag{17}$$

$$Y_l' = 1 \tag{18}$$

$$Y_k'' = 2(Y_0/n)^{1/2} \tag{19}$$

$$B_k = B_{n+1-k} = 2(Y_0/n)\sin\left[(k-1)\phi\right]\sin(k\phi)/\sin\phi \tag{20}$$

$$B_k' = 0 \tag{21}$$

where $l = 1,2,\cdots,n-1$, $k = 1,2,\cdots,n$, and ϕ is given by (13). Actually, values for ψ other than $\pi/2$ may be chosen, provided that the remaining equations are modified accordingly. However, this would unnecessarily complicate the circuit since B_k' would no longer be zero.

Note from (17)–(21) that the circuit has left-to-right symmetry, that $Y_1 = Y_{n-1} = Y_0$, and that $B_1 = B_n = 0$. Note also that if $\phi = 90°$, which can only occur if n is even, then (20) gives $B_k = 0$ for all k. Thus $n(90°)$ dividers/combiners do not contain any shunt susceptances.

Several examples of $n(\phi)$ dividers/combiners obtained from (14)–(21) are given in Fig. 4, where all port admittances are assumed to be unity, i.e., $Y_0 = 1$. When needed, shunt susceptances were realized by symmetric arrangements of transmission-line stubs. The 2(90°) case shown in the figure is the familiar branch-line four-port 3-dB quadrature hybrid [1]. The 4(90°) case is unique in that all its elements are identical quarter-wave transmission lines having normalized characteristic admittances of unity!

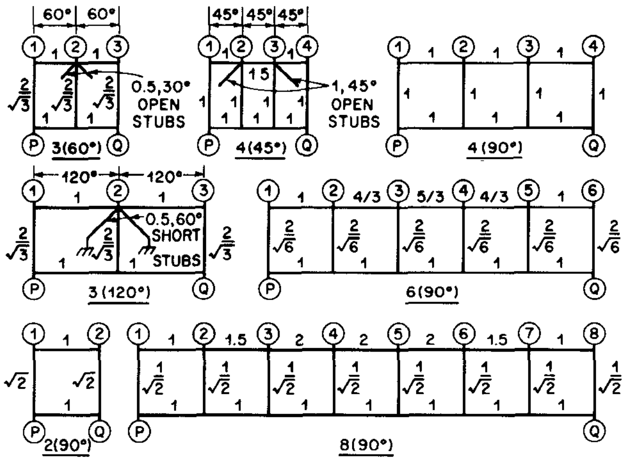

Fig. 4. Lossless branch-line realizations of some $n(\phi)$ dividers/combiners. All electrical lengths not shown are 90°. The numbers shown next to the lines are their characteristic admittances. All port admittances are unity.

IV. Interactions Among the Device Ports

A. Lossless Realizations

The interactions among the device ports, i.e., ports 1 through n, of the $n(\phi)$ dividers/combiners are represented by the matrix $\boldsymbol{R}$ defined in (5). It can be shown from (5) and (8) that if the network is lossless, i.e., if [1, p. 149]

$$\tilde{\boldsymbol{S}}^*\boldsymbol{S} = \boldsymbol{1} \tag{22}$$

which still satisfies (4b), then

$$\boldsymbol{R}^*\boldsymbol{R} = \boldsymbol{R}\boldsymbol{R}^* = \boldsymbol{1} - \boldsymbol{p}\tilde{\boldsymbol{p}}^* - \boldsymbol{q}\tilde{\boldsymbol{q}}^*. \tag{23}$$

This equation, together with (6)–(9), gives $\boldsymbol{R} = \boldsymbol{0}$ for $n = 2$, and $\boldsymbol{R} \neq \boldsymbol{0}$ for $n > 2$. Thus, except for the 2(90°) case, the device ports of the lossless $n(\phi)$ dividers/combiners shown in Fig. 4 are, in general, neither individually matched, nor pair-wise isolated.

No general formula could be found by the author that gives $\boldsymbol{R}$ as a function of $\boldsymbol{p}$ and $\boldsymbol{q}$. However, one can, of course, calculate $\boldsymbol{R}$ for each of the particular realizations given in Fig. 4. Some of these results are given below:

$$\boldsymbol{R}_{\substack{3(60°)\\3(120°)}} = 3^{-1}e^{\pm j2\pi/3}\begin{bmatrix} 1 & \mp 1 & 1 \\ \mp 1 & 1 & \mp 1 \\ 1 & \mp 1 & 1 \end{bmatrix} \tag{24}$$

$$\boldsymbol{R}_{4(90°)} = (-j/2)\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}. \tag{25}$$

In the 3(60°) and 3(120°) cases, ports 1, 2, and 3 have the same match and isolation of 9.5 dB. In the 4(90°) case, all ports are perfectly matched, and perfect isolation is obtained between ports 1 and 3 and between ports 2 and 4. The isolation between each of the remaining port pairs is 6 dB.

For the 6(90°) case, the worst match, which occurs at

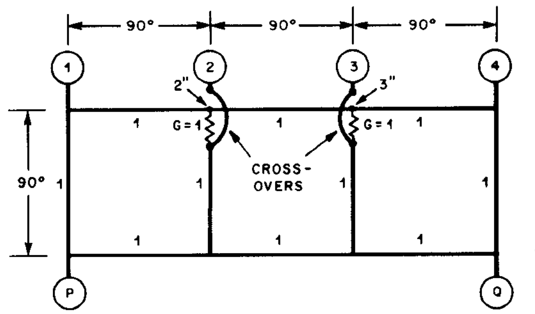

Fig. 5. A 4(90°) divider/combiner with matched and isolated device ports. All port admittances are unity.

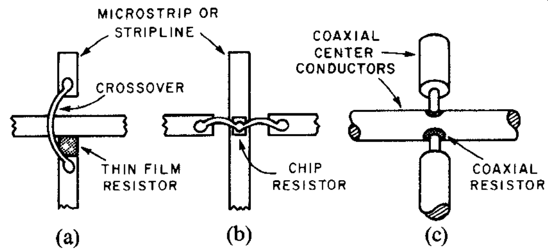

Fig. 6. Three practical realizations of the isolation resistors and the associated crossovers used in Fig. 5.

ports 3 and 4, is 8.0 dB, and the worst isolation, which occurs between ports 1 and 2 or between 5 and 6, is 4.9 dB. For the 8(90°) case, the worst match, which occurs at ports 3, 4, 5, and 6, is 6.0 dB, and the worst isolation, which occurs between ports 1 and 2 or between 7 and 8, is 4.7 dB.

B. Improving the Match and Isolation

In many applications, the device ports of dividers and combiners need to be matched and pair-wise isolated, i.e., the elements of $\boldsymbol{R}$ in (5) are required to be zero, or at least to have small values. This, for example, would eliminate undesirable oscillations which may exist when combining several one-port negative-resistance devices or two-port devices having an appreciable reverse gain. Also, when several power amplifiers are being combined, the match and isolation would result in graceful degradation of the output power when one or more of the amplifiers suffer arbitrary modes of failure [7].

It was shown by the author [8] that for n ports of a passive network to be matched and isolated from one another, at least n distinct resistors or terminations should be included in the network. Since ports P and Q in our $n(\phi)$ dividers/combiners can be considered as two terminations, then at least $n-2$ additional isolation resistors are needed to obtain the desired perfect match and isolation of the n device ports. In fact, realizations can be found where exactly $n-2$ such resistors are used, and where all the combining properties given in (5)–(10) are still fulfilled. These realizations are obtained from the lossless circuits in Fig. 4 by making an appropriate split at each of the intermediate ports, i.e., ports 2 through $n-1$, and inserting a resistor at each split. The simplest case is the 4(90°) divider/combiner with perfect match and isolation shown in Fig. 5. Various possible realizations of the required isolation resistors and the associated crossovers are shown in Fig. 6.

A 3(60°) divider/combiner with perfect match and isolation is shown in Fig. 7(a). A simplified circuit that gives 21.0 dB of match and isolation for all the device ports is shown in Fig. 7(b). Similar circuits with the same values of G hold for the 3(120°) case. Realizations found thus far by the author for other $n(\phi)$ dividers/combiners with perfect match and isolation are somewhat impractical, and thus will not be presented. However, some simplified circuits are shown in Fig. 8 for 6(90°) and 8(90°) dividers/combiners whose match and isolation, though not perfect, are

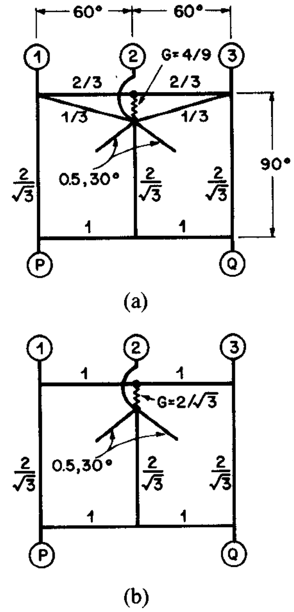

Fig. 7. A 3(60°) divider/combiner with (a) completely matched and isolated device ports, and (b) a simplified version with 21.0 dB of match and isolation for all device ports. All port admittances are unity.

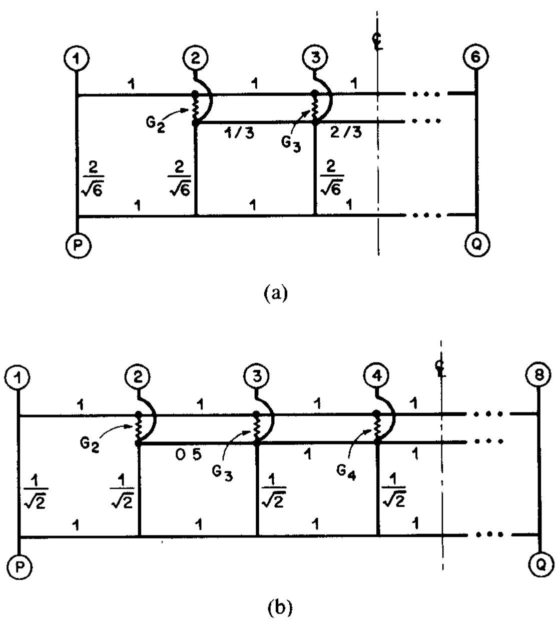

Fig. 8. (a) A 6(90°) divider/combiner, and (b) an 8(90°) divider combiner with improved match and isolation of the device ports. All lines are 90° in length, and all port admittances are unity. The values of the G's are given in Table I.

TABLE I
VALUES OF THE OPTIMUM ISOLATION CONDUCTANCES FOR THE 6(90°) AND 8(90°) DIVIDERS/COMBINERS OF FIG. 8

$n(\phi)$	Optimized Quantity	Optimum Conductances $G_2/G_3/G_4$	Worst Match (at ports)	Worst Isolation (between ports)
6(90°)	Match	1.302 0.740	26.6 dB (1,2,5,6)	11.1 dB (3-4)
	Match and Isolation	0.490 1.181	14.9 dB (2,5)	13.3 dB (2-3,2-5,3-4,4-5)
8(90°)	Match	∞ 0 0	20 dB (all ports)	3.7 dB (1-2,3-4,5-6,7-8)
	Match and Isolation	1.111 2.389 1.111	9.4 dB (3,6)	9.4 dB (4-5)

improved over those of their lossless counterparts in Fig. 4. The values of the normalized conductances, G's, of the isolation resistors needed to maximize the worst match alone, or the worst match and isolation are given in Table I. Note that the first optimization of the 8(90°) case actually contains no isolation resistors.

If the admittance of each of the device ports is Y_0, rather than unity as was assumed in this section, all values of G's should be multiplied by Y_0. The remaining elements of the various circuits are given by (17)–(20).

V. AN EXPERIMENT

A microstrip 4(90°) divider/combiner with matched and isolated device ports, Fig. 5, was built to operate at 3 GHz with 50-Ω port impedances. A 0.635-mm alumina substrate ($\epsilon_r \approx 10$) with a 50-Ω/square resistive tantalum–nitride film under a chrome–gold coating was used. Each of the required 50-Ω quarter-wave microstrip transmission lines had a width of 0.605 mm and a length of 10.2 mm, measured from center to center of the associated junctions. Each of the required 50-Ω isolation resistors was made of a 0.6-mm square of the resistive film as depicted in Fig. 6(a). Three ultrasonically bonded 51×18-μm gold ribbons were used to realize each of the crossovers.

The measured frequency responses of various key parameters are shown in Fig. 9. All ports were terminated in 50-Ω loads, except for the case labeled "with Γ = 1" where ports 1 through 4 were open-circuited. As expected in that case, essentially all the power entering port P emerged from port Q as indicated on the P–Q coupling curve on the top part of the figure. The middle part of the figure shows that the return loss of port P changed only slightly as the terminations of ports 1 through 4 changed from Γ=0 to Γ=1.

Depending on the criterion employed, the operating bandwidth ranges from 5 to 10 percent. The shift of the center frequency from 3 GHz to about 2.8 GHz is attributed to junction effects, which were not considered in the design.

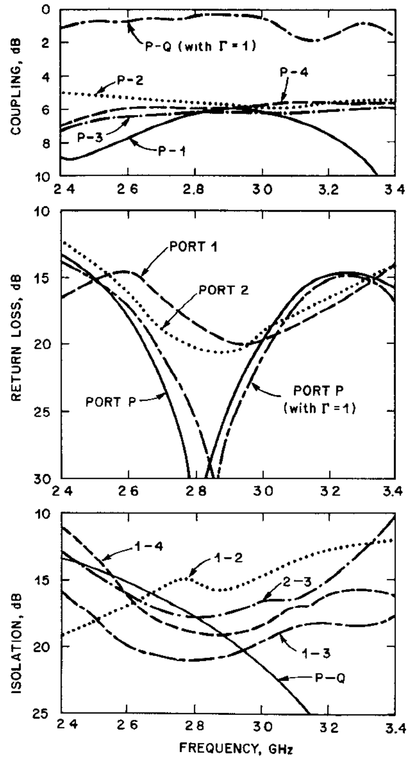

Fig. 9. Experimental results of a microstrip version of the 4(90°) hybrid power divider/combiner of Fig. 5. All ports are terminated in matched loads, except the case labeled "with Γ = 1" where ports 1 through n are open-circuited.

VI. CONCLUSIONS

A new class of planar, multiport power dividers/combiners, which generalizes the familiar 3-dB quadrature hybrid, has been presented. They are capable of combining an arbitrary number of identical one-port or two-port devices while maintaining a match at the input port, even if the devices are mismatched. An external port is provided to absorb the reflections from the devices. The new dividers/combiners, which provide equal, n-way power split with equal phase delay ϕ between successive ports, were referred to as *$n(\phi)$ dividers/combiners.* With this terminology the 3-dB quadrature hybrid can be named "2(90°) divider/combiner." Generalizations to obtain unequal power splits or phase delays are possible, but were not presented.

Lossless branch-line realizations were given in which the device ports generally have imperfect match and isolation. Realizations containing isolation resistors were also given that yield improved match and isolation. In fact, realizations with perfect match and isolation were given for the 3(60°), 3(120°), and 4(90°) cases. An experimental 4(90°) divider/combiner of that type was built and shown to give acceptable performance over a bandwidth of 5 to 10 percent. This is typical of all $n(\phi)$ dividers/combiners discussed in this paper.

Beside their use in combining a number of power devices, $n(\phi)$ dividers/combiners can be used for feeding multi-element antennas where progressive, equal phase delay of the signal is desired. Another interesting mixer application is mentioned in [9].

ACKNOWLEDGMENT

The author thanks F. A. Pelow for his help in building the divider/combiner, and in performing the measurements in the experiment described in Section V.

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On Solving Waveguide Junction Scattering Problems by the Conservation of Complex Power Technique

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Abstract—**Normal mode expansions are used to mode match the tangential electric field at the transverse junction of two cylindrical waveguides. Instead of mode matching the tangential magnetic field the principle of conservation of complex power is invoked and leads, without a matrix inversion, to an expression for the junction's input admittance matrix, as seen from the smaller guide. Simple matrix algebra and the reciprocity theorem then provide the generalized scattering matrix of the two-port (with higher order modes included). It is also shown that the solution satisfies the continuity condition for tangential magnetic field in the junction's aperture. Numerical results are given for parallel plate waveguides with TEM, TE_1, and TM_1 incident fields, numerical convergence being achieved with about ten modes in the smaller waveguide.**

I. INTRODUCTION

THEORETICAL and experimental studies of electromagnetic scattering at waveguide junctions have occupied the attention of numerous researchers for several decades. The *variational method* has provided quite accurate dominant mode solutions to a wide variety of waveguide discontinuity problems [1], [2]. Since the advent of fast digital computers interest has shifted to *numerically oriented techniques* due to their broad scope. However, among other restrictions [3]–[6], their range of application is generally limited to two-dimensional problems. A rather complete listing of the related literature is given in the dissertation [7] of one of the authors of this paper.

Following a circuit theory approach, Sharp [8] has presented an exact solution for the admittance matrix of a T-junction of rectangular waveguides. The size of this matrix, which includes propagating and evanescent modes of all ports, makes this approach inefficient when the scattering matrix of the junction or of a single port is of interest. Wexler [9], in dealing with the transverse junction of cylindrical waveguides, begins with continuity equations of transverse $\boldsymbol{E}$ and $\boldsymbol{H}$, the field being expressed in terms of waveguide modes, and obtains a solution for the mode coefficients. Though the generalized form of reciprocity in n-port junctions, as derived in [7], could be employed to solve the reverse problem, i.e., when the incidence direction

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